Criterion Modeling in the Control Problems

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***Abstract* — The use of the criterion modeling for solving problems of optimal control of dynamic systems has been suggested in the article and the method of determining the matrix of similarity criteria of the control law has been developed. The comprehensive approach to the reduction of the complexity degree of optimal control problem has been developed, which is represented as the problem of optimization of nonlinear programming, when the matrix approach of conversion of factors and improvement of mathematical model are used. Matrix formation of optimal similarity criteria is implemented by using mathematical models of the transitive system of relative units.**

***Index Terms* — criterion modeling, nonlinear optimization, per unit system, relative units, optimal control, similarity theory.**

I. Introduction

In complex dynamical systems there is a tendency of transition from natural functioning problems of dynamic systems to much more difficult problems of optimal control [1]. Due to the increasing capabilities of computer science and microtechnology, which are used in control systems, the solution of problems is technically available.

The efficiency of optimal control is known to depend on the accuracy and adequacy of mathematical models. Automation process of the optimal control of dynamical systems is characterized in general by multipronged approaches to the formation of mathematical apparatus. It is possible to improve the efficiency of the system operation of optimal control by using the system approach and the integrated methodological basis at all stages of solving the problem of optimal control, starting with the formation of a mathematical model and ending with the analysis of the optimal solution and its practical implementation. The use of generalized methods of similarity theory and modeling is highly productive in this aspect [2], the criterion method in particular [3], which is based on their foundation and can be used at all levels of this problem solving. These models permit to generalize the results of optimal control and extend them to a number of similar phenomena.

II. Formation of Goals and Problem Statement

The object under investigation in this work is process of optimal control of dynamical systems. The subject of the research is criterion modeling in optimal control problems.

The goal is the improvement of system operation efficiency of optimal control by using similarity theory and criterion modeling and its development.

The following problems are solved to achieve the goal:

1. To analyze the possibilities of using the similarity theory and criterion modeling in the problems of the synthesis of control and optimization laws.
2. To develop criterion models of the optimal control of dynamic systems.
3. To develop the methods and software of operation of criterion models in optimal control.

Realization this problems will enable to form the scientific and systematic concept of the essence of criterion modeling for optimal control problems. The development of criterion models using generalized methods of similarity theory and simulation is actual in the optimal control of dynamic systems.

In general, the problem of the optimal control of the dynamic process consists in the identification of steering function that minimizes the loss function presented as a quality criterion under appropriate constraints [4]. In this case, the steering function of the system must generate a controllable action, in which the transition of the system is in its optimal state that is accompanied by a minimal loss of resources, energy or time.

For this type of problems the necessary conditions of extremum were formulated by Pontryagin in the form of a continuous and discrete "maximum principle" [5], which in general consists in reducing the optimal control problem to the calculation of the maximum of Hamilton [4].

Considering the further possibility of practical realization of the optimal solutions in the systems of automatic control in terms of similarity theory and criterion modeling, the optimal control law can be formulated like this [3]:

$$U(t) = -\pi\, x(t), \tag{1}$$

where $U(t)$, $x(t)$ are vector of the operating parameter and vector of the operated parameter respectively; π is a matrix of feedback factors provided in the form of a matrix of similarity criteria.

The criterion processing of optimal control permits to present mathematical models of the process as a criterion notation in per unit systems, which makes it possible to establish the mathematical relationship between variables of an optimal control problem and the similarity criteria, which, in turn, determine the weight of the relevant parameters of the mathematical model in optimality criteria, and to summarize the research results and extend them to a number of similar phenomena.

In addition, the criterion dimensionless form of the control law record (1) makes it possible to analyze the obtained results for sensitivity and proportionality, using the mathematical apparatus of the criterion analysis [6, 7], and also allows to predict the dynamics of the process, using the

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means of criterion prediction [8] and fuzzy logic [9].

Thus, the use of generalized methods of the similarity theory and simulation of the criterion method, in particular, is quite effective for the optimal control of dynamic systems.

III. The Development of Mathematics to the Problem

At the stages of mathematical model forming, computation and practical realization of the solution (the optimal control law), it is necessary to use per unit system because of the fundamental singularity display of optimal control problems. The criterion method makes it possible to use such different per unit systems as heuristic, derivative, transit, criterion, differential, semiotic and signum in the control theory and it significantly extends the capabilities of the criterion method and the area of the practical realization of the optimal solution [3, 10].

The use of criterion models in optimal control permits to monitor the analytic connection between control process parameters and parameters of system elements where this process takes place. Individual characteristics and system properties are researching as well as the synthesis of its variants.

In this case, the optimal control problem is reduced to solving a nonlinear optimization problem of such kind:

$$y = \sum_{i=1}^{m_1} A_i \prod_{j=1}^{n} x_j^{\alpha_{ji}} \to \min ;$$

$$g_k = \sum_{i=m_k+1}^{m_{k+1}} A_i \prod_{j=1}^{n} x_j^{\alpha_{ji}} \le 1 ;$$

$$k = \overline{1,\, p} ; \quad x_j > 0 , \qquad (2)$$

where $y(x)$ is generalized engineering-and-economical performance; x_j is variable parameters of system; n is number of variables; m is total amount of mathematical model summand; p is a number of limitations; A_i, α_{ij} is the constant coefficient that determines system properties.

The system of equations for the terms of the objective function and limitations (2) follows from transition to the variables of the dual problem of criterion modeling [3], using the transitive per unit system:

$$\prod_{j=1}^{n} x_j^{\alpha_{ji}} = \frac{\pi_i \cdot y_{\min}}{A_i} , \; i = \overline{1,\, m_1} ;$$

$$\prod_{j=1}^{n} x_j^{\alpha_{ji}} = \frac{\pi_i}{A_i \sum_{r=m_k+1}^{m_{k+1}} \pi_r} , \; i = \overline{m_k+1,\, m_{k+1}} , \; k = \overline{1,\, p} . \qquad (3)$$

In the general case, the problem of the optimal control of complex dynamic systems under research has a high degree of complexity that defines the ways of this problem solving. The use of iteration methods of sequential search for extremum in different per unit systems [3] consequently leads to the accumulation of a computational error. Here in the article, there is suggested the approach of bringing the problem of a high degree of complexity into a canonical form in a transitive per unit system and this problem solving by criterion modeling.

The degree of the complexity of the criterion modeling problem depends on the number of summands of objective function and the number of variables, that is $s = m - n - 1$ and it exerts influence on the formation of the vectors of dependent and independent variables of the dual problem of criterion modeling.

After finding the logarithm, the system of equations (3) in a matrix form is presented as:

$$\begin{vmatrix} \alpha_{11} & \dots & \alpha_{n1} & -1 & \gamma_{11} & \dots & \gamma_{1s} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \alpha_{1m_1} & \dots & \alpha_{nm_1} & -1 & \gamma_{m_1 1} & \dots & \gamma_{m_1 s} \\ \alpha_{1m_1+1} & \dots & \alpha_{n(m_1+1)} & 0 & \gamma_{(m_1+1)1} & \dots & \gamma_{(m_1+1)s} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \alpha_{1m} & \dots & \alpha_{nm} & 0 & \gamma_{m1} & \dots & \gamma_{ms} \end{vmatrix} \times$$

.

$$\times \begin{vmatrix} \ln(x_1) \\ \dots \\ \ln(x_n) \\ \ln(y_{\min}) \\ \ln(\pi_1) \\ \dots \\ \ln(\pi_s) \end{vmatrix} = \begin{vmatrix} \ln(\pi'_1 / A_1) \\ \dots \\ \ln(\pi'_{m_1} / A_{m_1}) \\ \ln\left(\pi'_{m_1+1} \middle/ A_{m_1+1} \cdot \sum_{r=m_1+1}^{m_2} (\pi_r) \right) \\ \dots \\ \ln\left(\pi'_m \middle/ A_m \cdot \sum_{r=m_p+1}^{m} (\pi_r) \right) \end{vmatrix} , \qquad (4)$$

where $\begin{cases} \gamma_{ij} = -1, & i = j; \\ \gamma_{ij} = 0, & i \ne j; \end{cases} \quad \begin{cases} \pi'_i = 1, & i \le s; \\ \pi'_i = \pi_i, & i > s. \end{cases}$

When the degree of the complexity of the problem is positive $s > 0$, the similarity criteria are divided into dependent $\pi_1 \dots \pi_s$ and independent $\pi_{s+1} \dots \pi_m$.

The use of the inverse matrix of system (4) relates parameters of direct and dual problems of criterion modeling to one another. The element complement method of the inverse matrix of objective function $d(B)$ and the refinement of proposed model are used if the degree of the complexity of the problem is positive.

$$B = \begin{vmatrix} \beta_{11} & \dots & \beta_{1m} \\ \dots & \dots & \dots \\ \beta_{n1} & \dots & \beta_{nm} \\ \beta_{p_0 1} & \dots & \beta_{p_0 m} \\ \beta_{p_1 1} & \dots & \beta_{p_1 m} \\ \dots & \dots & \dots \\ \beta_{p_s 1} & \dots & \beta_{p_s m} \end{vmatrix} \xrightarrow{d(B)} B' = \begin{vmatrix} \beta_{11} & \dots & \beta_{1m} \\ \dots & \dots & \dots \\ \beta_{n1} & \dots & \beta_{nm} \\ \beta'_{p_0 1} & \dots & \beta'_{p_0 m} \\ \beta_{p_1 1} & \dots & \beta_{p_1 m} \\ \dots & \dots & \dots \\ \beta_{p_s 1} & \dots & \beta_{p_s m} \end{vmatrix} .$$

The method of transition $d(B)$ consists in identifying the factors $c_i,\ i=\overline{1,\ s}$ that satisfy the conditions

$$\prod_{j=1}^{m}\left(\frac{-\beta_{p_0 j}-c_i\cdot\beta_{p_i j}}{A_j}\right)^{\beta_{p_i j}}=1,\quad i=\overline{1,\ s}\ ,$$

and determining the values of the similarity criteria which are correlated with a minimum value of the objective function $y_{\min}$:

$$\beta'_{p_0 j}=-\beta_{p_0 j}-\sum_{i=1}^{s}c_i\cdot\beta_{p_i j},\ j=\overline{1,\ m}\ .$$

The minimum value of the objective function and corresponding values of arguments are determined in this way:

$$y_{\min}=\prod_{j=1}^{m}\left(\frac{\beta'_{p_0 j}}{A_j}\right)^{\beta'_{p_0 j}}\cdot\prod_{k=1}^{p}\left(\sum_{r=m_k+1}^{m_{k+1}}\left(\beta'_{p_0 r}\right)\right)^{\sum_{r=m_k+1}^{m_{k+1}}\left(\beta'_{p_0 r}\right)},$$

$$x_i=\prod_{j=1}^{m}\left(\frac{\beta'_{p_0 j}}{A_j}\right)^{\beta_{ij}}\cdot\prod_{k=1}^{p}\left(\sum_{r=m_k+1}^{m_{k+1}}\left(\beta'_{p_0 r}\right)\right)^{\sum_{r=m_k+1}^{m_{k+1}}\left(\beta'_{ir}\right)},i=\overline{1,\ n}\ .$$

A decrease of the degree of complexity is achieved through the introduction of the extra factors $c_i,\quad i=\overline{1,\ s}$ in the row $\left|\beta_{p_0 1}\ \ \dots\ \ \beta_{p_0 m}\right|$ of the matrix B .The vector of optimal similarity criteria is defined by the expression:

$$\pi=-\left|\beta'_{p_0 1}\ \ \dots\ \ \beta'_{p_0 m}\right|=\left|-\beta'_{p_0 1}\ \ \dots\ \ -\beta'_{p_0 m}\right|.$$

The transformed objective function of the mathematical model of a zero power of complexity is determined from the inverse matrix:

$$A'=\left(B'\right)^{-1}.$$

In this case, the coefficients γ_{ij} of the matrix are as follows:

$$\begin{cases}\gamma_{ij}=\gamma_{ij}+c_i,\ j\le n+1;\\ \gamma_{ij}=0,\ j>n.\end{cases}$$

Thus, the generated canonical problem of optimal control is as follows:

$$y=\sum_{i=1}^{m_1}A_i\prod_{j=1}^{n}x_j^{\alpha_{ji}}\prod_{q=1}^{s}x_{n+q}^{\gamma_{qi}+c_q}\rightarrow\min\ ;$$

$$g_k=\sum_{i=m_k+1}^{m_{k+1}}A_i\prod_{j=1}^{n}x_j^{\alpha_{ji}},\ k=\overline{1,\ p}\ ;\ \ x_j>0\ ;\ \ x_{n+q}>0\ .$$

The number of variables of the model parameters is one less by a unit than the total number of its terms. Problem solving of a canonical type in criterion modeling is realized in the software package “Search and analysis of optimal solutions” as well as problems of a high degree of complexity. This software package integrates various methods of per unit system in optimal control.

IV. Conclusions and Prospects for the Further Use of the Research Results

Thus, the possibility of increasing the efficiency of optimal control by means of its realization in the similarity theory and criterion modeling it has been considered in this article. To that end, the following conclusions were obtained.

1. The method for determining the matrix of similarity criteria of the law of optimum control has been developed. It should be noted some of the benefits of using the criterion modeling: the use of the common methodological basis and systematic approach to solving the problem, the application of mathematical models of various per unit systems through which there is established a direct connection between the variables of a direct and dual problem of criterion programming, the ability to solve the problem of high complexity without iterative methods of the sequential search of extremum.

2. To solve problems of the control of complex dynamical systems there has been developed the comprehensive approach of bringing the problem of high degree of complexity into a canonical form in transitive per unit system of criterion modeling by finding the logarithm. On the basis of the proposed criterion models of transitive per unit system there has been developed a method for solving nonlinear programming problem of a high degree of complexity by converting ratios and refining the mathematical model of the matrix method.

3. Software for the search and analysis of optimal solutions has been developed, which realize the proposed models and methods of criterion modeling in optimal control.

The further investigation of subject matter will be directed towards complex variable domain to expand the area of solutions of problem of the optimal control for dynamic systems. Since models of transitive per unit system are restricted by the positive domain of function parameters when searching feasible solutions of extreme problems in connection with finding the logarithm, the research will be conducted to enhance the practical field of transitive per unit system research in the polynomial problem class with negative criteria of similarity or the implicit competitive effect of optimal control function.

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